Sverdrup and nonlinear dynamics of the Pacific South Equatorial Current

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(JPO, in press)

- **Previous work on the Pacific equatorial momentum balance has implied near-linearity.**
- **New observations of near-but-off-equatorial zonal current spanning the basin do not agree with linear Sverdrup calculations.**
- **A feature only resolved in satellite wind stresses partially reconciles discrepancies between observed and Sverdrup currents.**
- **A tropical OGCM (Gent and Cane) shows the importance of non-linear terms (vorticity advection and friction) in explaining the differences between observed and Sverdrup currents.**

Previous depictions of the Sverdrup circulation have shown only a weak circulation near the Pacific equator:

(JPO)

JANUARY 1987 WILLIAM S. KESSLER AND BRUCE A. TAFT 121

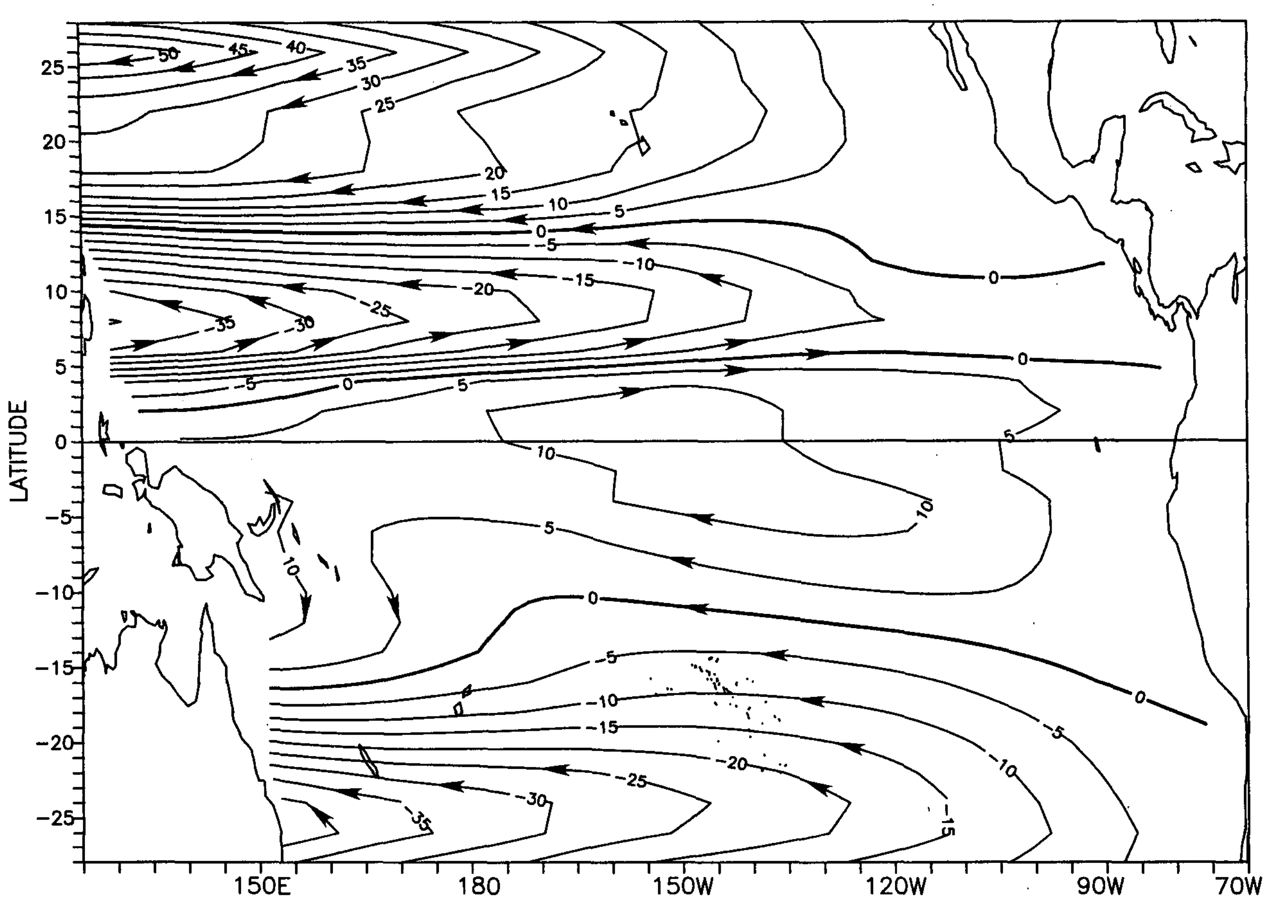


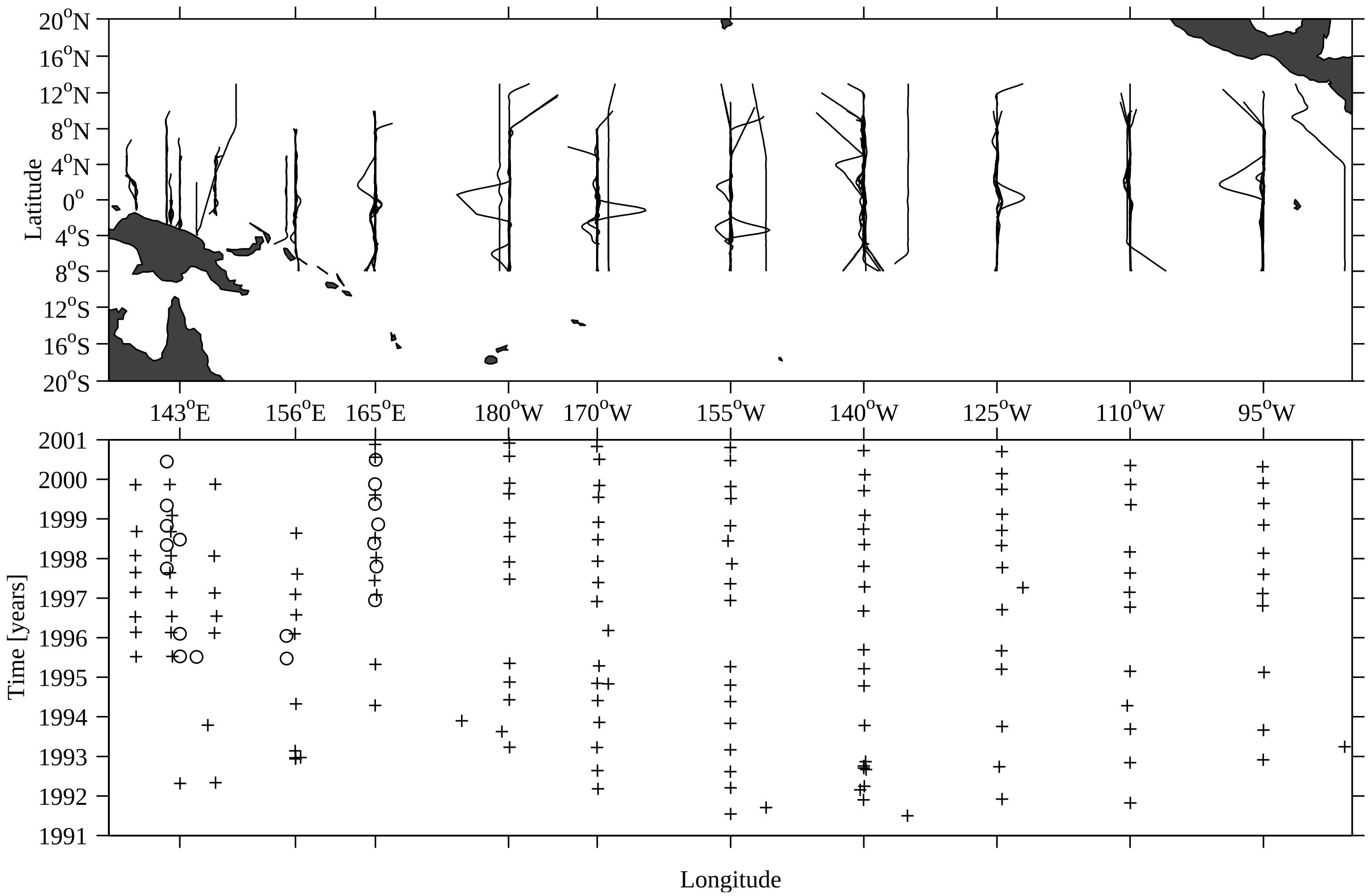
FIG. 26. Streamlines of volume transport (each contour represents 5 Sv) computed from the Sverdrup relation using the average wind stress from 1979 through 1981.

Why is the Sverdrup circulation so weak?

Possible reasons:

1. It really is that way.
2. The wind is wrong.
3. Sverdrup dynamics are too simple for this situation.

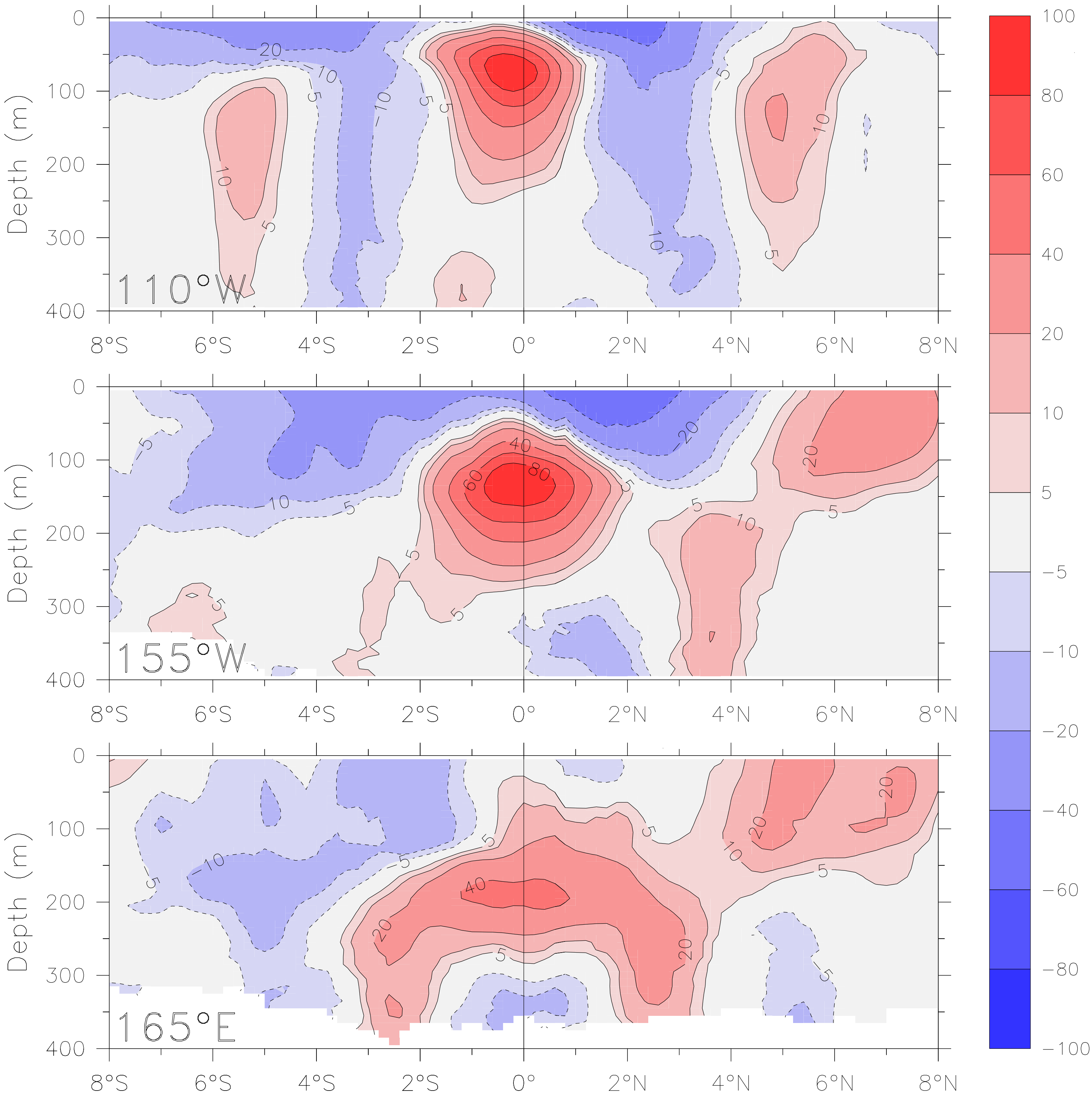
CTD / ADCP data distribution (Johnson et al. 2002)



Top: Ship tracks for the 172 meridional sections used in this study.
Bottom: Equator-crossing times of these sections. 1991-2001 shown by “+”; 1985-1990 shown by “o”, with 10 years added for compactness.

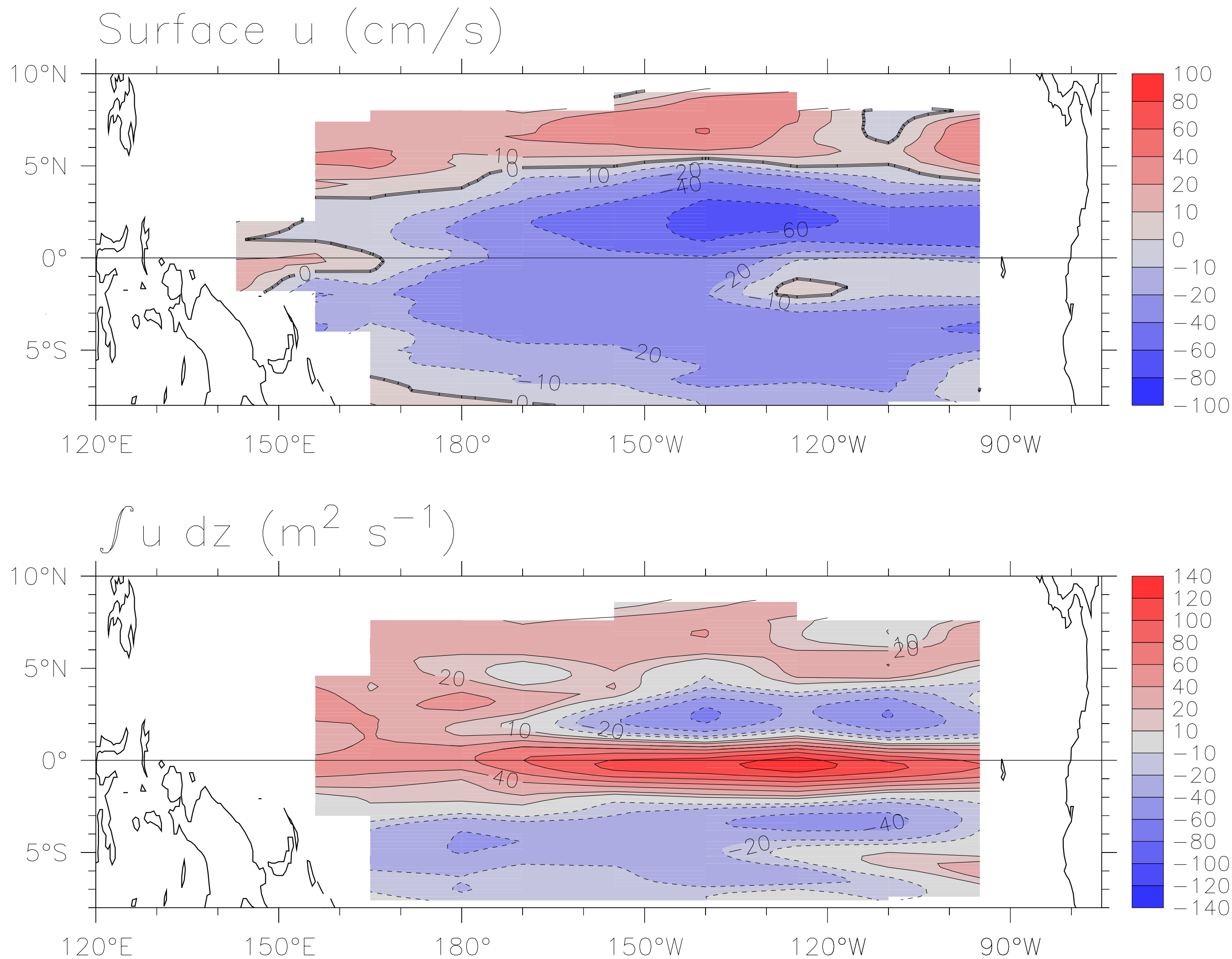
Mean zonal current (cm s^{-1})

Johnson ADCP data

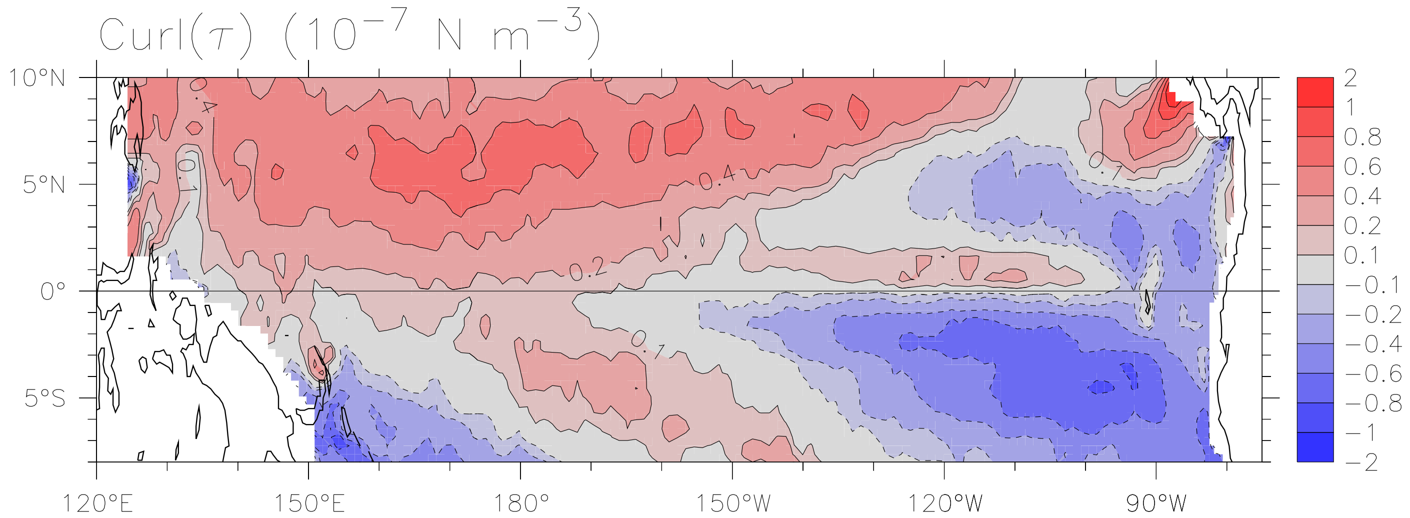


Surface and vertically−integrated zonal current

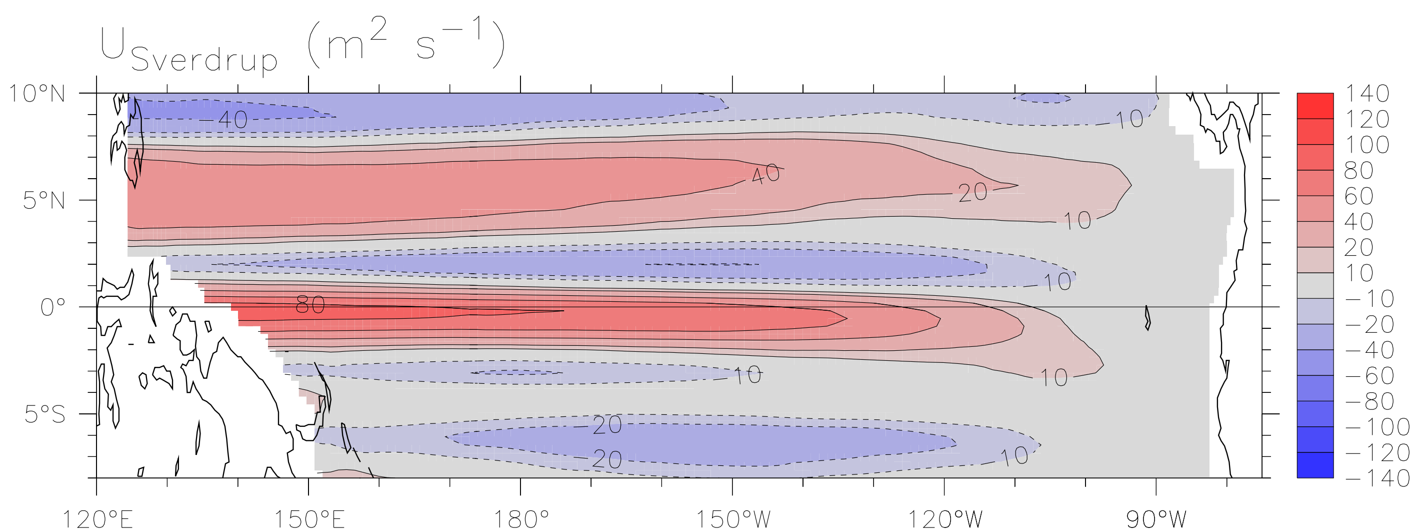
Johnson ADCP data

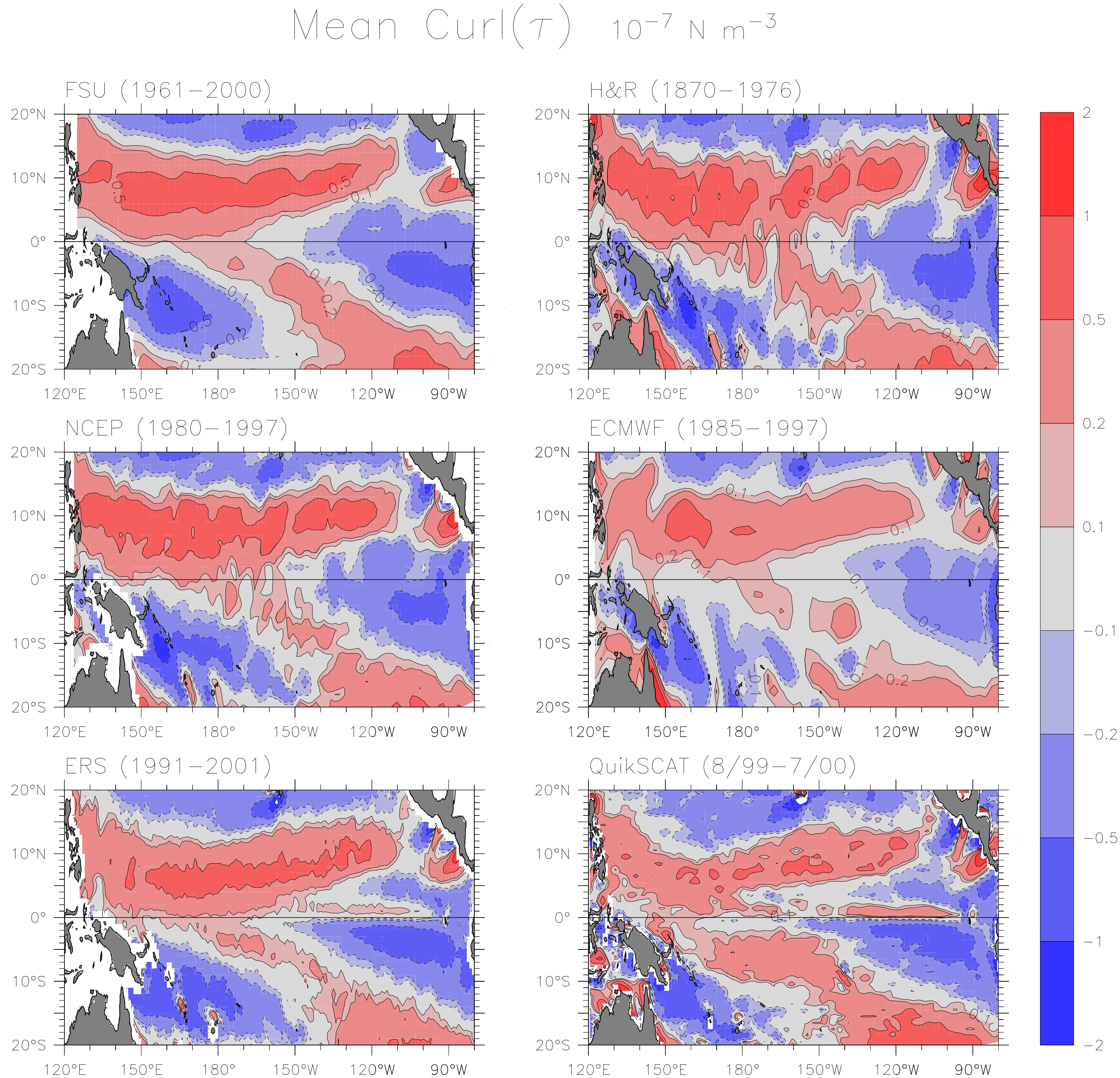


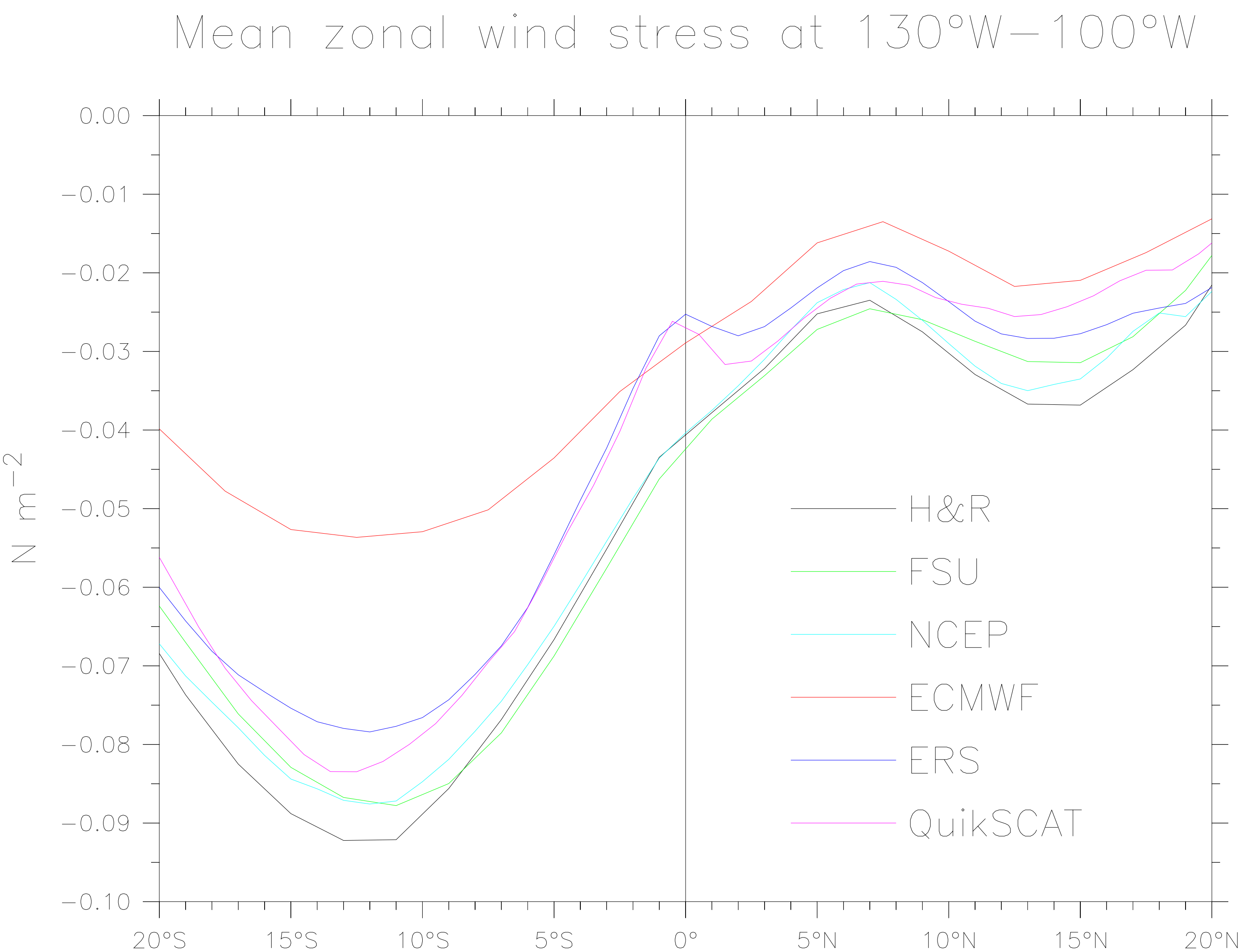
Mean Curl(τ) (ERS winds 1991–2000)

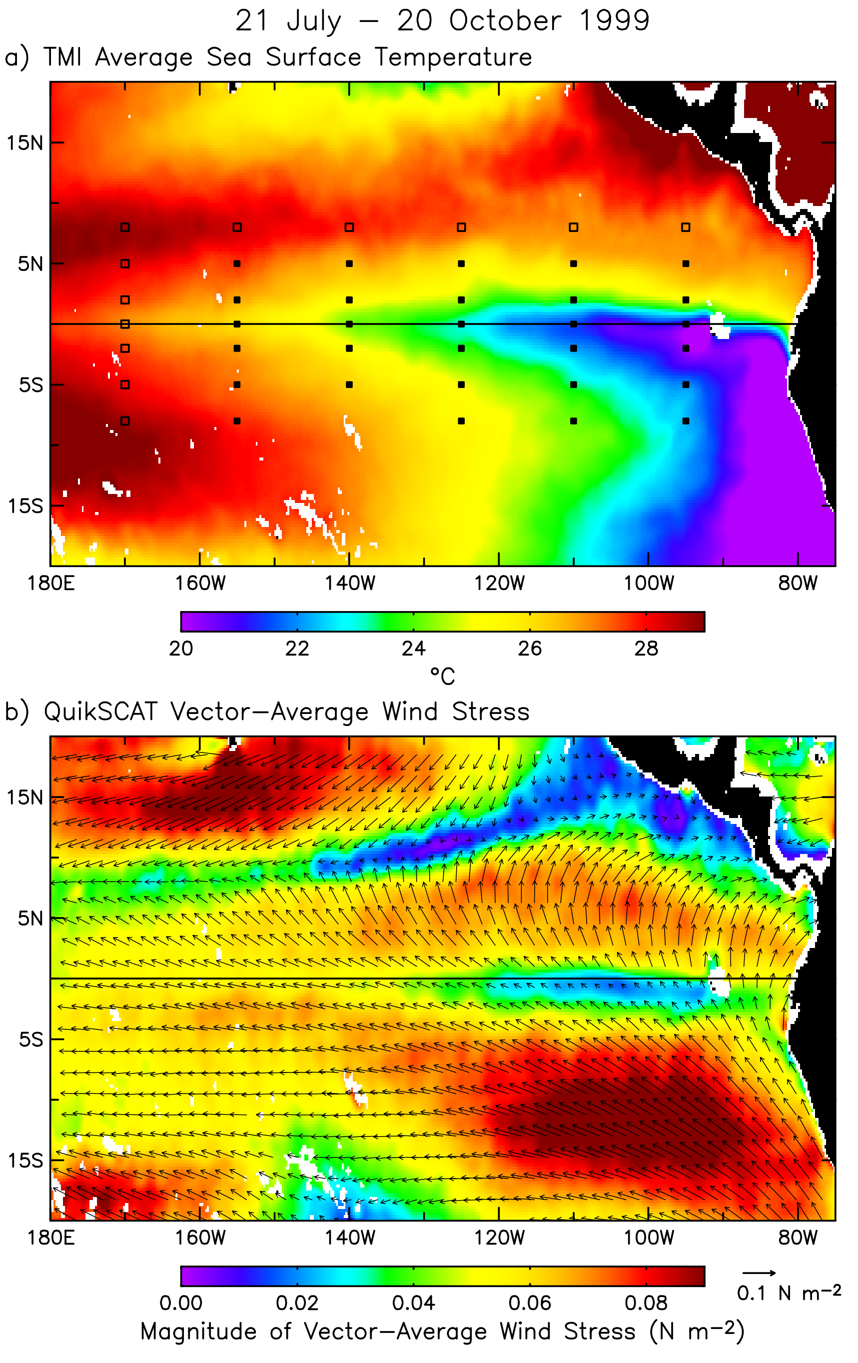


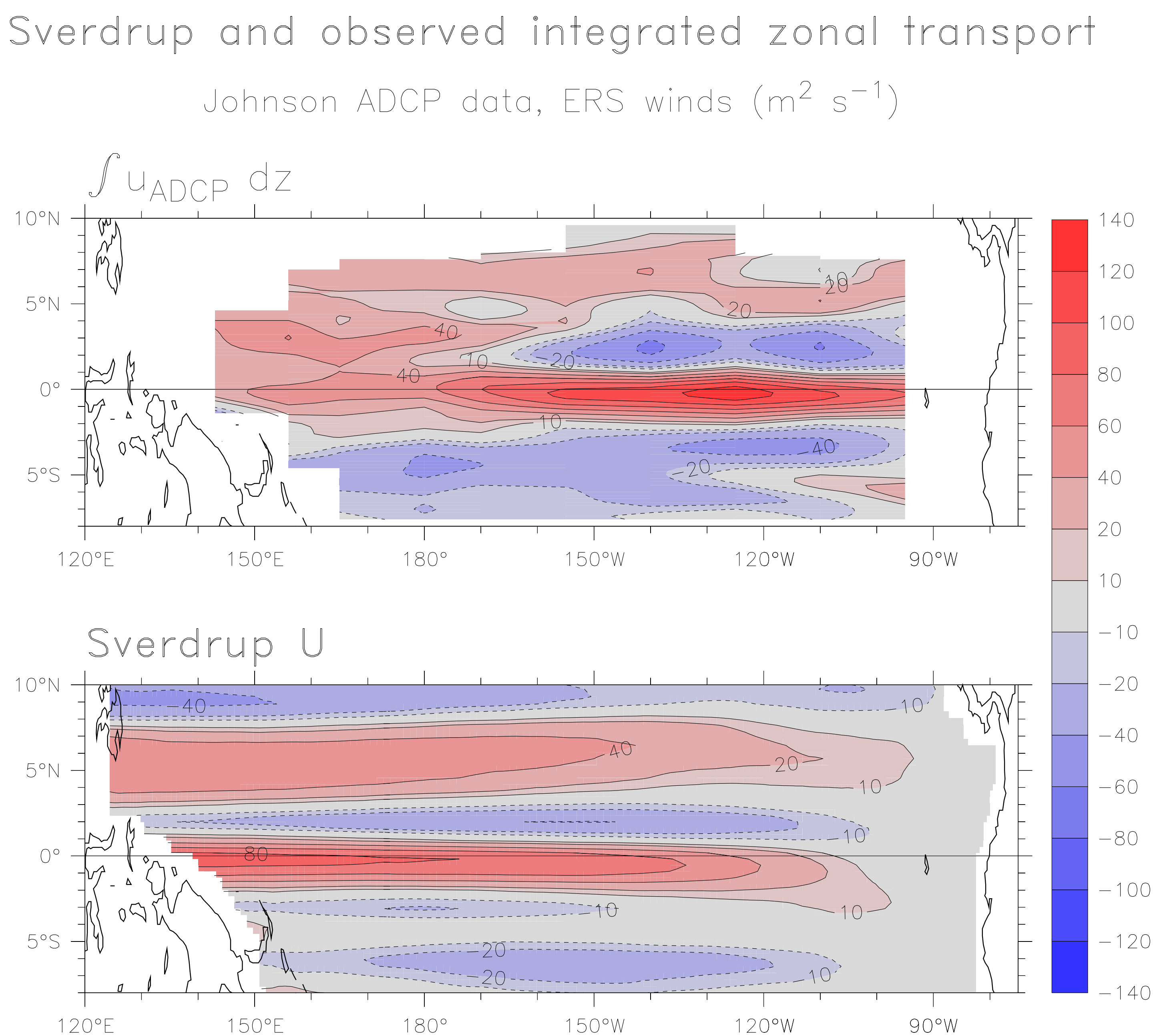
$$\left.\begin{aligned} -fV &= -P_x + \tau^x \\ fU &= -P_y + \tau^y \\ U_x + V_y &= 0 \end{aligned}\right\} \qquad \begin{aligned} \beta V &= Curl(\tau) \\ U &= -\frac{1}{\beta}\int_{EB}^{x} Curl(\tau)_y \, dx + U_{EB}(y) \end{aligned}$$




Mean Curl(τ) 10^{-7} N m^{-3}
FSU (1961–2000)
H&R (1870–1976)
NCEP (1980–1997)
ECMWF (1985–1997)
ERS (1991–2001)
QuikSCAT (8/99–7/00)
20°N
10°N
0°
10°S
20°S
120°E
150°E
180°
150°W
120°W
90°W
2
1
0.5
0.2
0.1
−0.1
−0.2
−0.5
−1
−2


Mean zonal wind stress at 130°W–100°W
0.00
−0.01
−0.02
−0.03
−0.04
−0.05
−0.06
−0.07
−0.08
−0.09
−0.10
N m−2
20°S
15°S
10°S
5°S
0°
5°N
10°N
15°N
20°N
H&R
FSU
NCEP
ECMWF
ERS
QuikSCAT


21 July — 20 October 1999
a) TMI Average Sea Surface Temperature
15N
5N
5S
15S
180E
160W
140W
120W
100W
80W
20
22
24
26
28
°C
b) QuikSCAT Vector—Average Wind Stress
0.00
0.02
0.04
0.06
0.08
Magnitude of Vector—Average Wind Stress (N m−2)
0.1 N m−2



Sverdrup and observed integrated zonal transport
Johnson ADCP data, ERS winds (m2 s−1)
∫ uADCP dz
Sverdrup U
10°N
5°N
0°
5°S
120°E
150°E
180°
150°W
120°W
90°W
140
120
100
80
60
40
20
10
−10
−20
−40
−60
−80
−100
−120
−140

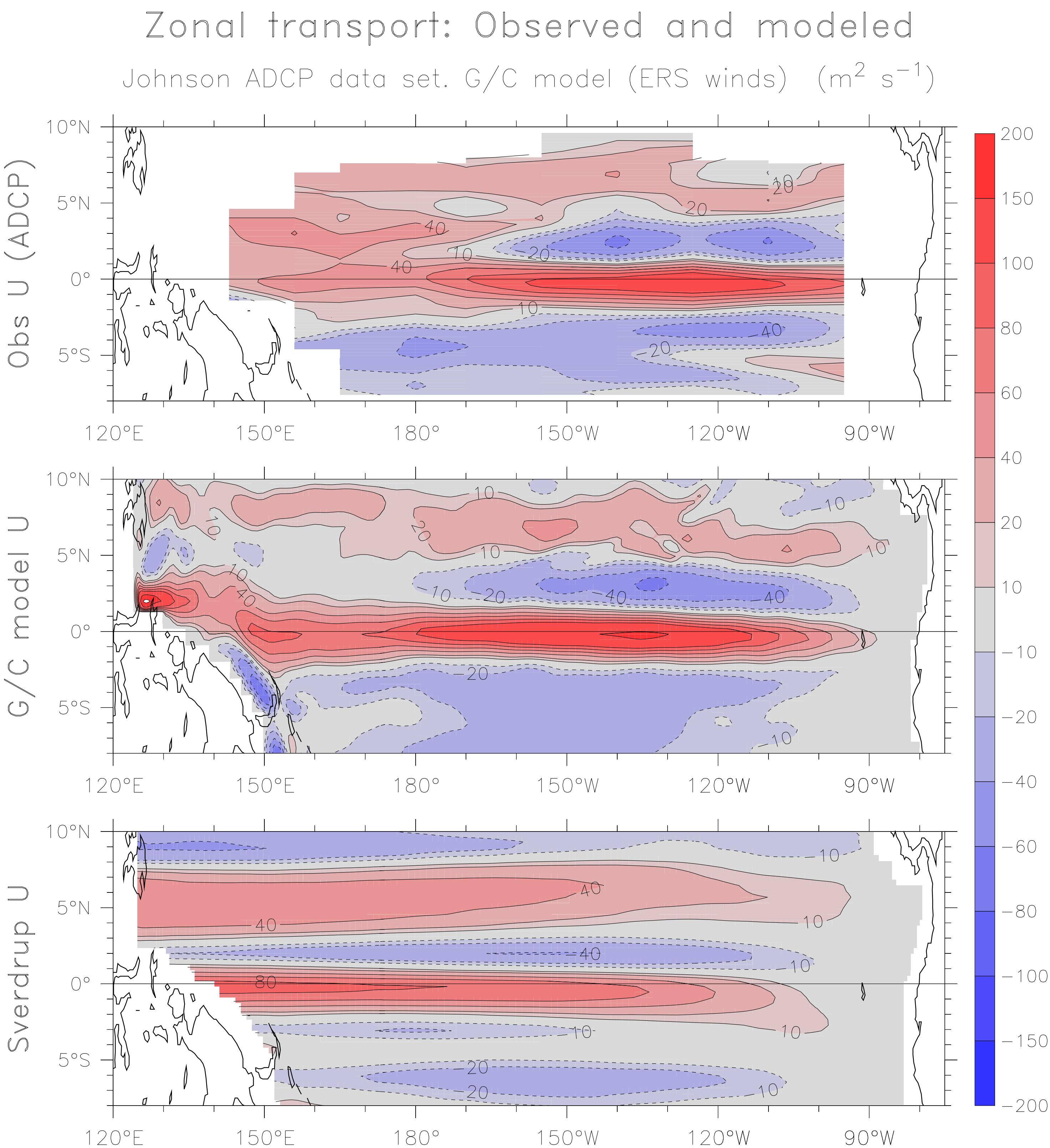
Gent and Cane (1989) OGCM

- **Sigma-coordinate OGCM.**
  → Sigma here has nothing to do with density!

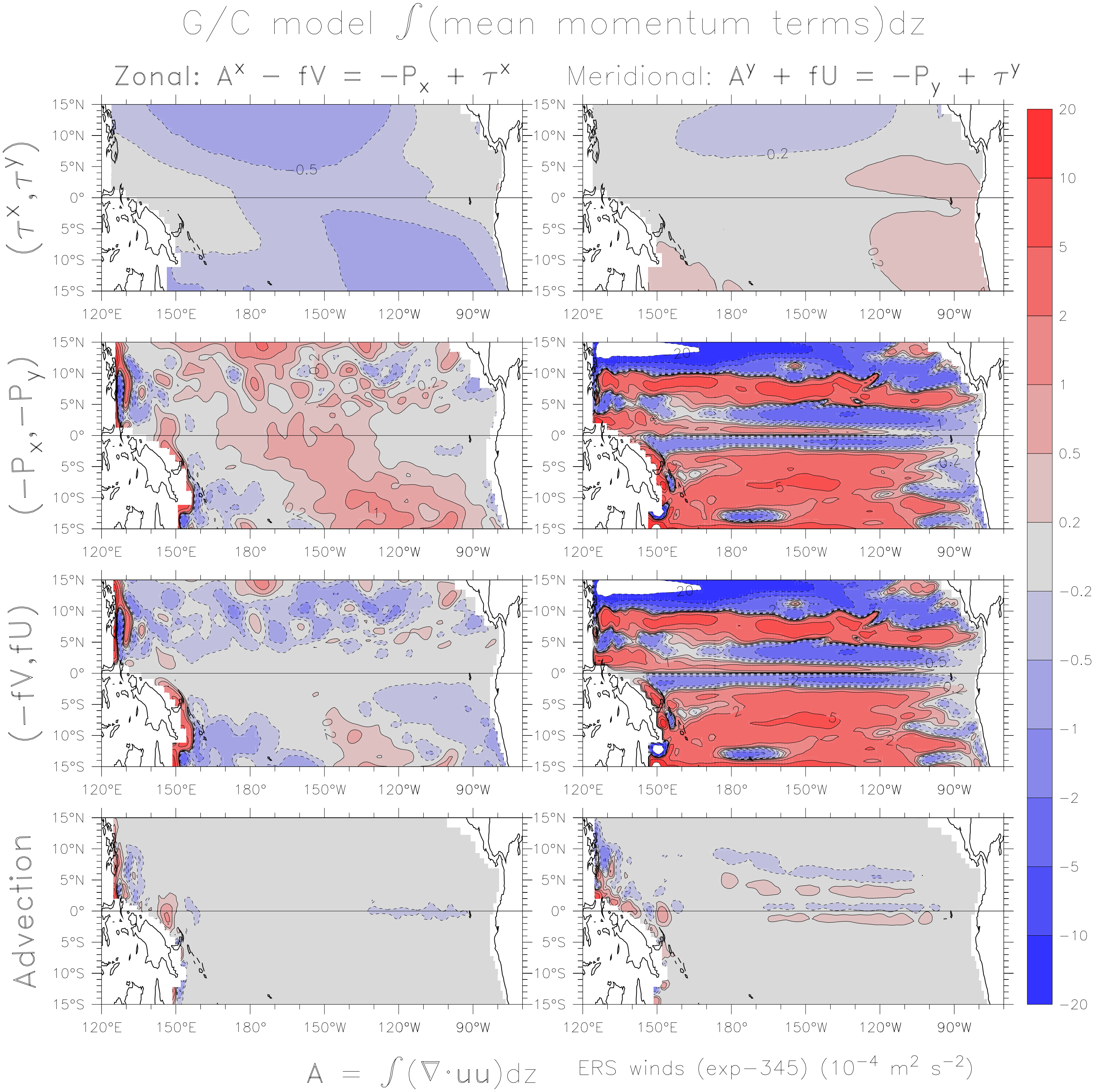
- **The entire model is the (stratified) upper layer of a reduced gravity ocean; the mean depth of the model is about 400m.**
  Within this upper layer, there is an explicit (Niiler-Kraus) mixed layer and 9 sigma layers.

- **The domain is the tropical Pacific from 30°S-30°N, with realistic east-west boundaries. The northern and southern boundaries are solid walls with relaxation to Levitus poleward of 20° to suppress coastal Kelvin waves.**
  Horizontal resolution: $\Delta y \approx 40\,\mathrm{km}$, $\Delta x \approx 100\,\mathrm{km}$.

- **The model is forced with an average annual cycle of 1991-2001 ERS winds (and ISCCP clouds) for 10 years. All results shown are an average over model year 10. The model has reached near-equilibrium at this time (as shown by 40 year runs).**



Zonal transport: Observed and modeled
Johnson ADCP data set. G/C model (ERS winds) (m^2 s^{-1})
Obs U (ADCP)
G/C model U
Sverdrup U
10°N
5°N
0°
5°S
120°E
150°E
180°
150°W
120°W
90°W
200
150
100
80
60
40
20
10
−10
−20
−40
−60
−80
−100
−150
−200



G/C model ∫(mean momentum terms)dz
Zonal: $A^x - fV = -P_x + \tau^x$
Meridional: $A^y + fU = -P_y + \tau^y$
(τ^x, τ^y)
$(-P_x, -P_y)$
$(-fV, fU)$
Advection
15°N
10°N
5°N
0°
5°S
10°S
15°S
120°E
150°E
180°
150°W
120°W
90°W
20
10
5
2
1
0.5
0.2
−0.2
−0.5
−1
−2
−5
−10
−20
$A = \int(\nabla\cdot uu)dz$
ERS winds (exp−345) (10^{-4} m^2 s^{-2})

Diagnosing the role of the nonlinear terms

The vertically-integrated, time-mean momentum equations can be written:

$$A^x - fV = -P_x + \tau^x + F^x \qquad (1a)$$

$$A^y + fU = -P_y + \tau^y + F^y \qquad (1b)$$

where upper case symbols indicate vertically-integrated quantities, $A = (A^x, A^y) = \int \nabla \bullet uu\, dz$ are the advective terms and $F = (F^x, F^y)$ are the (combined) friction terms. Time means are taken after forming products such as A. The vertically-integrated mean continuity equation is:

$$U_x + V_y = 0 \qquad (2)$$

One way to *diagnose* the role of the advective and friction terms is to rearrange (1) so the advective and friction terms appear as analogs to forcing terms; that is, to define a generalized stress τ^*

$$\tau^* \equiv \tau + \tau' + \tau'' , \text{ where } \tau' \equiv -A \text{ and } \tau'' \equiv F .$$

Equations (1) then are rewritten

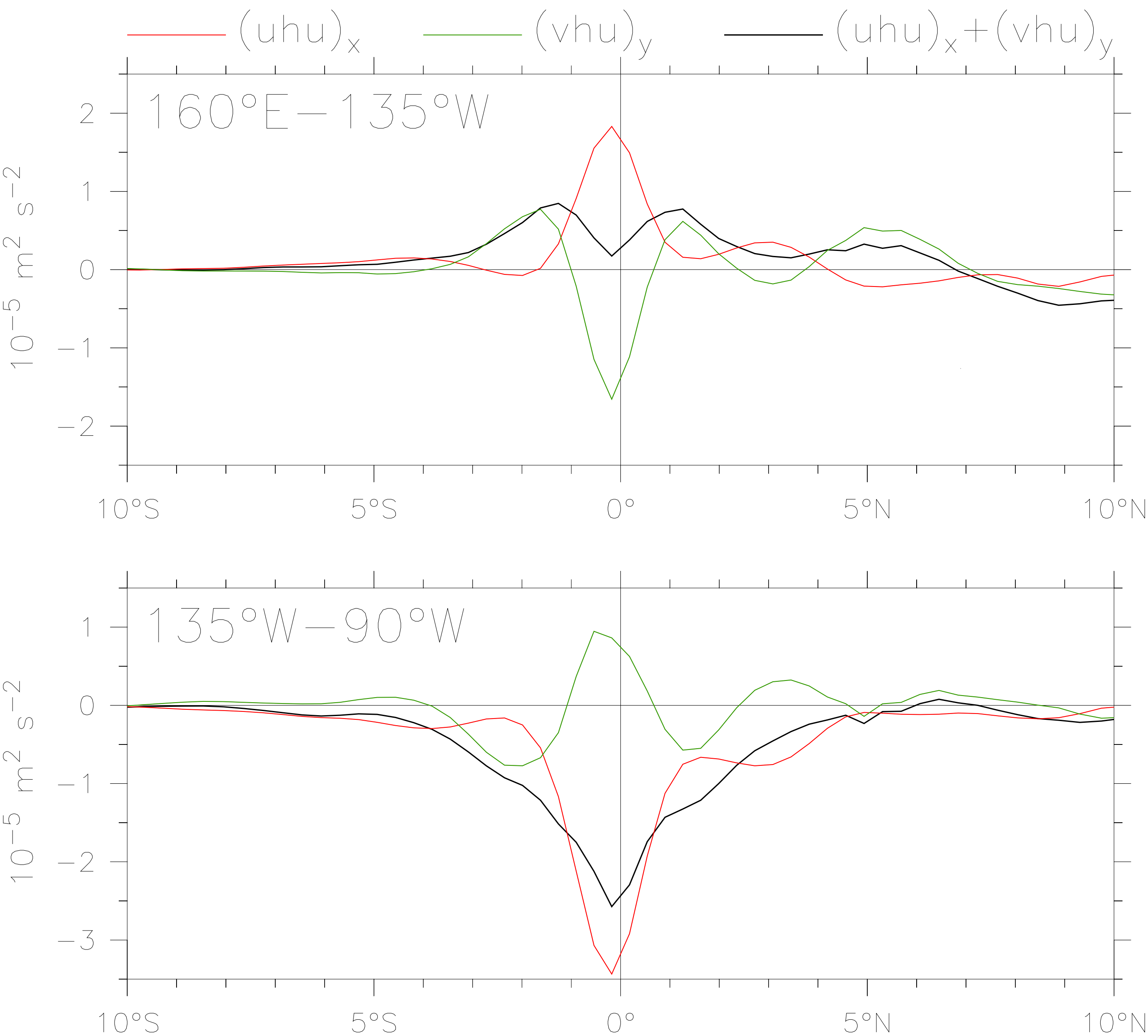
$$-fV = -P_x + \tau^{*x} , \qquad fU = -P_y + \tau^{*y} \qquad (3)$$

which have the same form as the linearized (Sverdrup) set. Taking the curl of (3) leads to a Sverdrup-like balance, with τ replaced by τ^*, in which the effects of the advective and friction terms are evaluated through their modification of the vorticity:

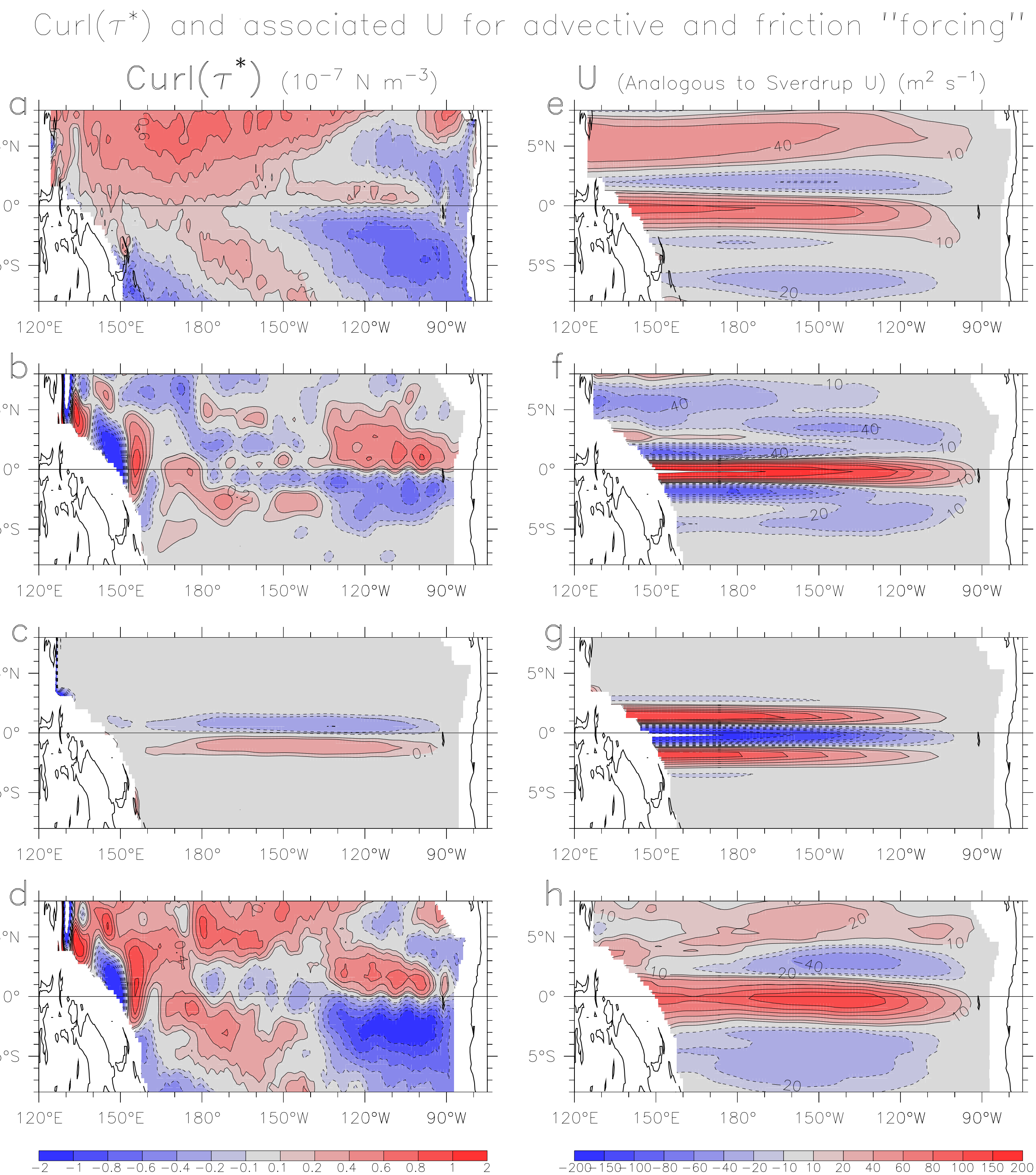
$$\beta V = Curl(\tau^*) , \qquad U = -\frac{1}{\beta}\int_{EB}^{x} Curl(\tau^*)_y\, dx + U_{EB}(y) \qquad (4,5)$$

- (3) is just a *rearrangement* of (1).
- (4) and (5) will be used to show that the importance of the advective and friction terms comes through their derivatives, which have quite different spatial patterns than the terms themselves.

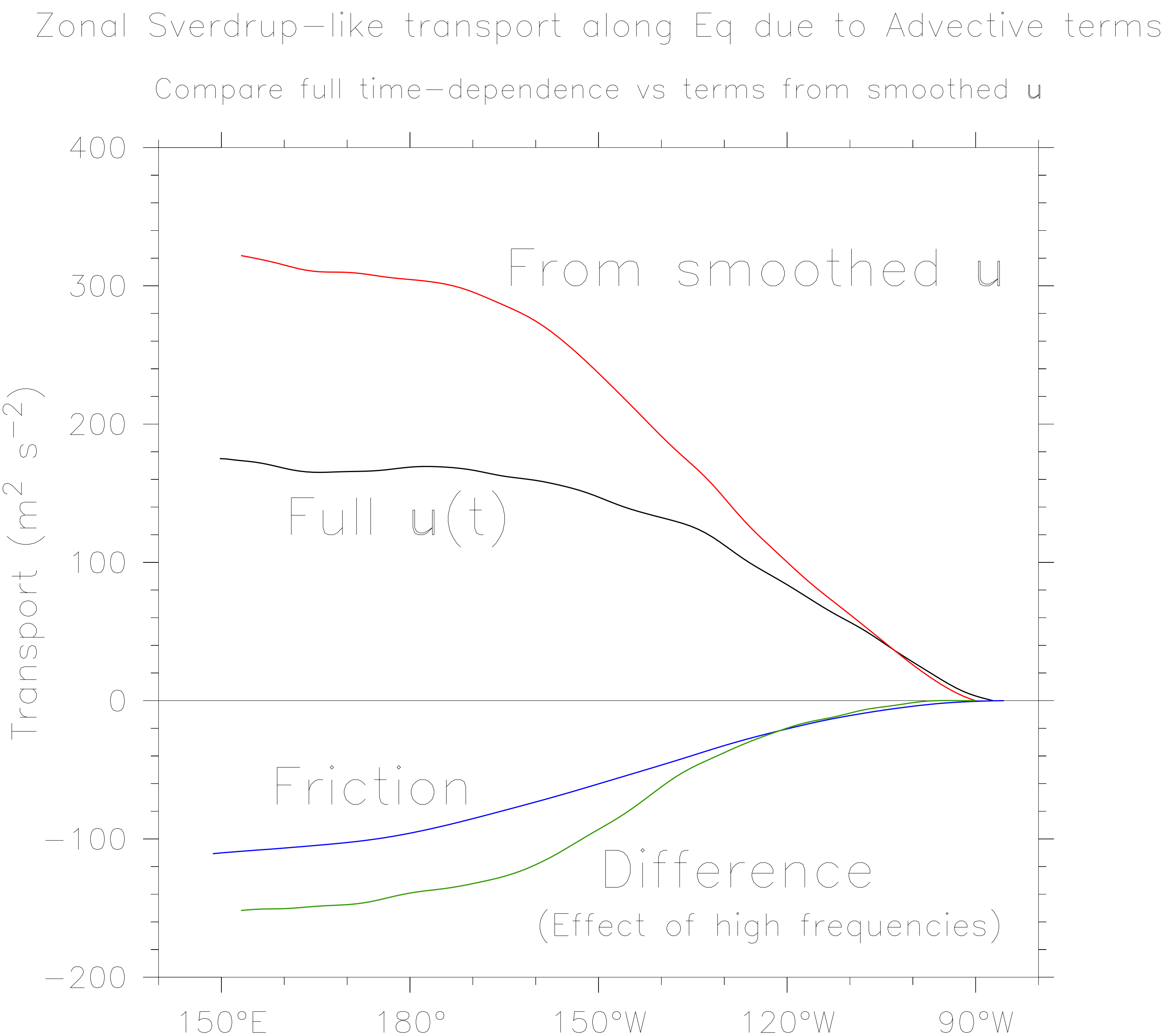
Divergence of zonal momentum flux



Exp–345 (Vertically integrated).



Curl(τ^*) and associated U for advective and friction "forcing"
Curl(τ^*) (10^{-7} N m^{-3})
U (Analogous to Sverdrup U) (m^2 s^{-1})
τ = Wind Stress
τ' = $-\int(\nabla\cdot uhu)dz$
τ'' = $\int$(Friction)dz
τ^* = Total
a
b
c
d
e
f
g
h
120°E
150°E
180°
150°W
120°W
90°W
5°N
0°
5°S
−2 −1 −0.8 −0.6 −0.4 −0.2 −0.1 0.1 0.2 0.4 0.6 0.8 1 2
−200 −150 −100 −80 −60 −40 −20 −10 10 20 40 60 80 100 150 200



Zonal Sverdrup-like transport along Eq due to Advective terms
Compare full time-dependence vs terms from smoothed u
Transport (m^2 s^{-2})
400
300
200
100
0
−100
−200
150°E
180°
150°W
120°W
90°W
From smoothed u
Full u(t)
Friction
Difference
(Effect of high frequencies)

Conclusion

1. Linear Sverdrup calculations using either ship or reanalysis wind stresses indicate very weak currents in the equatorial Pacific compared with directly observed zonal currents.

2. Only satellite scatterometer wind stresses resolve a strip of positive curl north of the equator (due to air-sea interaction) that contributes significantly to the Sverdrup transports. It is essential to use a realistic wind product.

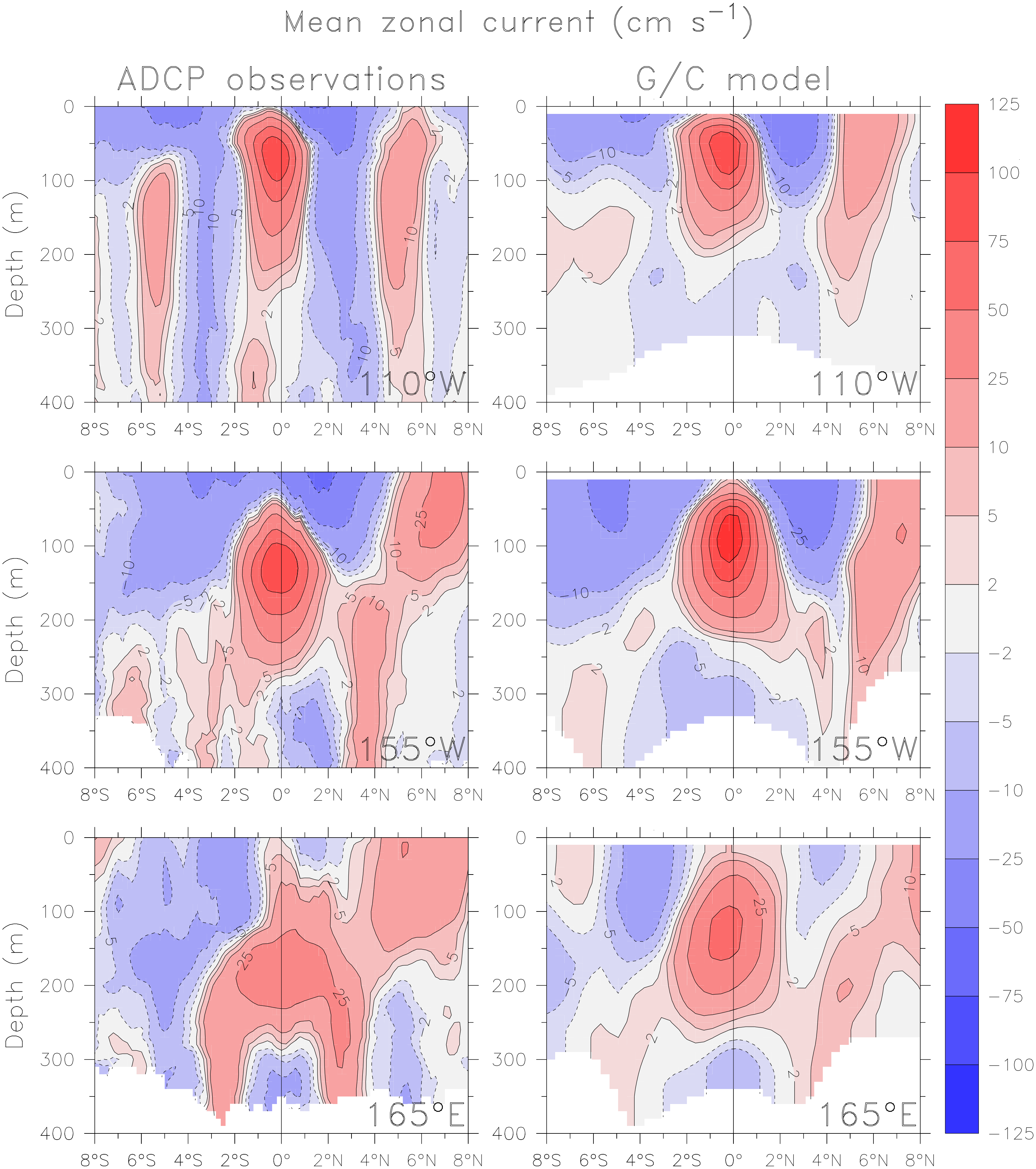
3. Diagnosis of nonlinear terms using an OGCM shows that:

   - Although the model momentum balance is nearly linear, its currents are not Sverdrupian. ➟ Vorticity balance.
   - Eastward advection of vorticity in the EUC doubles the strength of the zonal currents (both eastward and westward) in the central Pacific.
   - Friction damps the currents in the west, producing their central Pacific maximum.

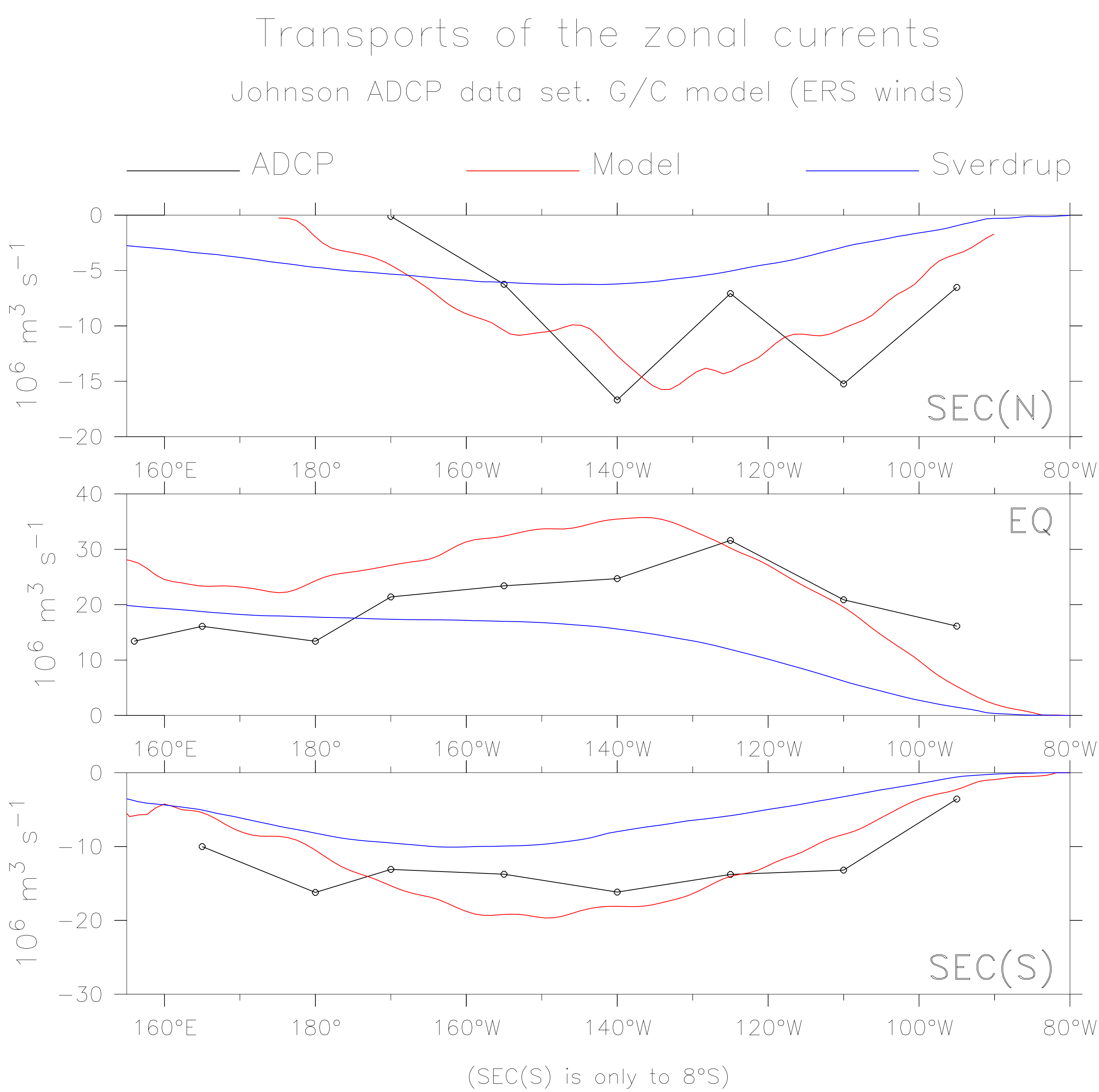
4. TIW act through advection to damp the eastward equatorial flow. Although total advection strengthens the flow, the high-frequency part reduces the effect by about half.
   This suggests that:
   a. The annual cycle of TIW damps the EUC during Jul-Feb.
   b. The absence of TIW during El Niño implies stronger eastward transport. Does this contribute to east Pacific warming?

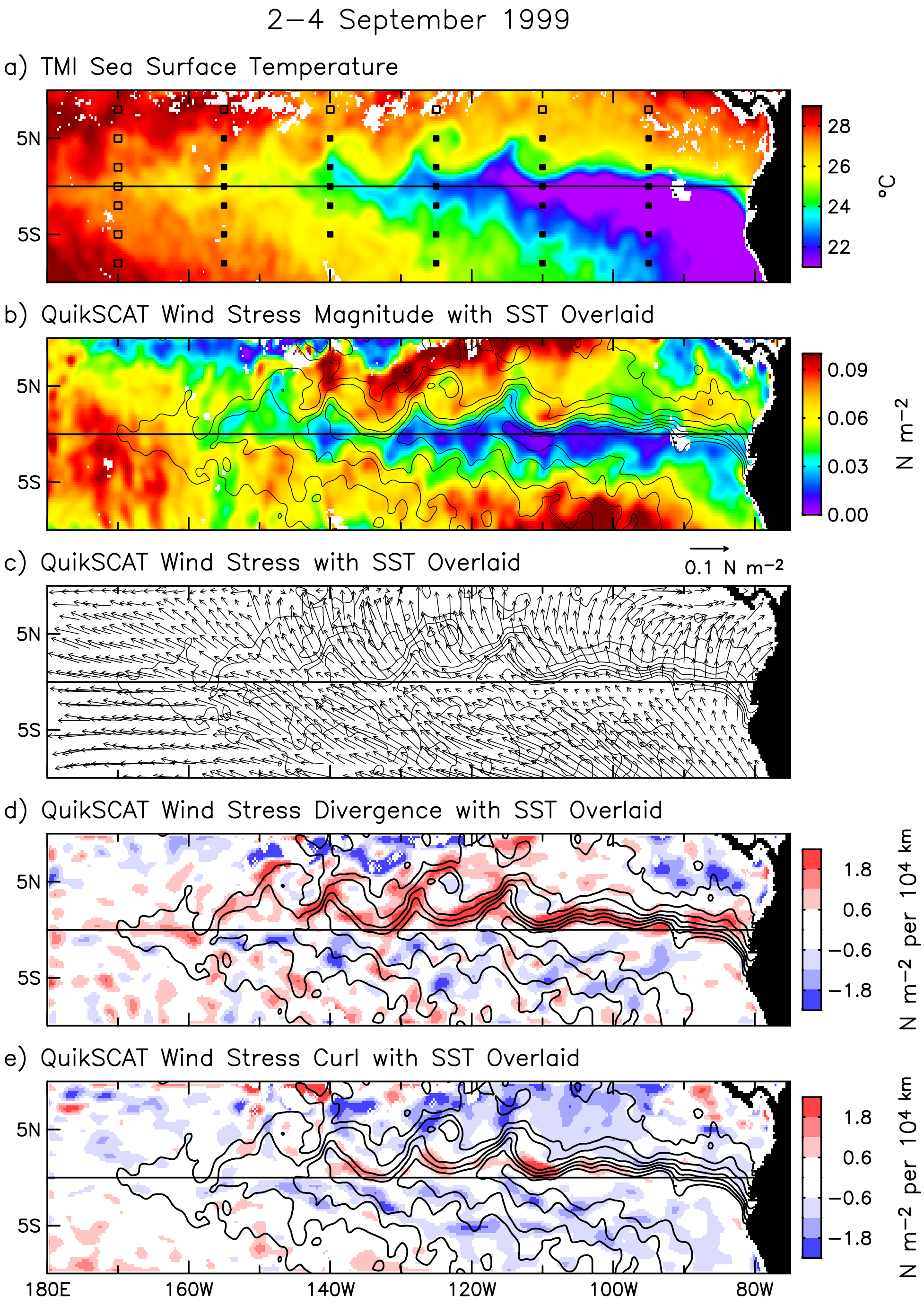
All figures at: http://www.pmel.noaa.gov/~kessler ➟ Latest Talk

Extra slides .....



Mean zonal current (cm s−1)
ADCP observations
G/C model
Depth (m)
110°W
155°W
165°E
8°S 6°S 4°S 2°S 0° 2°N 4°N 6°N 8°N
0 100 200 300 400
125 100 75 50 25 10 5 2 −2 −5 −10 −25 −50 −75 −100 −125


Transports of the zonal currents
Johnson ADCP data set. G/C model (ERS winds)
ADCP
Model
Sverdrup
SEC(N)
EQ
SEC(S)
10^6 m^3 s^{-1}
0
−5
−10
−15
−20
40
30
20
10
−30
160°E
180°
160°W
140°W
120°W
100°W
80°W
(SEC(S) is only to 8°S)


2–4 September 1999
a) TMI Sea Surface Temperature
b) QuikSCAT Wind Stress Magnitude with SST Overlaid
c) QuikSCAT Wind Stress with SST Overlaid
0.1 N m−2
d) QuikSCAT Wind Stress Divergence with SST Overlaid
e) QuikSCAT Wind Stress Curl with SST Overlaid
5N
5S
°C
28
26
24
22
N m−2
0.09
0.06
0.03
0.00
N m−2 per 10⁴ km
1.8
0.6
−0.6
−1.8
180E
160W
140W
120W
100W
80W

The boundary condition $\boldsymbol{U_{EB}(y)}$ in the integral for the Sverdrup zonal transport is often assumed to be zero, which is only true for a meridionally-oriented eastern boundary. When $\boldsymbol{Curl(\tau)}$ is non-zero at a tilted boundary, the Sverdrup relation implies flow normal to the boundary unless there was a corresponding zonal transport $\boldsymbol{U_{EB}}$ to make the total boundary flow exactly alongshore. This required value of $\boldsymbol{U}$ along the coast is the eastern boundary condition for the zonal integral.

This B.C. can be found using the Sverdrup streamfunction

$$\psi = \frac{1}{\beta}\int_{x_e(y)}^{x} Curl(\tau)\,dx \ , \quad V = \psi_x \ , \ U = -\psi_y$$

where $\boldsymbol{x_e(y)}$ is the longitude of the boundary at each latitude. The boundary condition for (A1) is ψ = constant, no matter what the boundary slope, since the no-normal flow condition precludes any ψ contours from intersecting the coast. The meridional derivative of (A1) gives the complete expression for $\boldsymbol{U}$ (using Liebniz' Rule):

$$U = -\psi_y = -\frac{1}{\beta}\left(\int_{x_e(y)}^{x} Curl(\tau)_y\,dx \ - \ d[x_e(y)]/dy\; Curl(\tau)\big|_{x=x_e(y)}\right)$$

where the first term on the right hand side is the contribution to $\boldsymbol{U}$ from interior wind forcing, and the second term is the value of $\boldsymbol{U}$ on the boundary ($\boldsymbol{U_{EB}}$). $\boldsymbol{d[x_e(y)]/dy}$ in that term is the boundary slope, which is zero for a meridional coast and positive clockwise.

Flux-form advective terms in Gent/Cane model

Flux form obtained by:

$h\cdot$(momentum equations) + $\vec{u}\cdot$(continuity equation).

The combined advection terms are thus:

$$h(\vec{u}\cdot\nabla\vec{u}) + \vec{u}(\nabla\cdot h\vec{u}) = \nabla\cdot(\vec{u}\vec{u}h)$$

Writing these terms out:

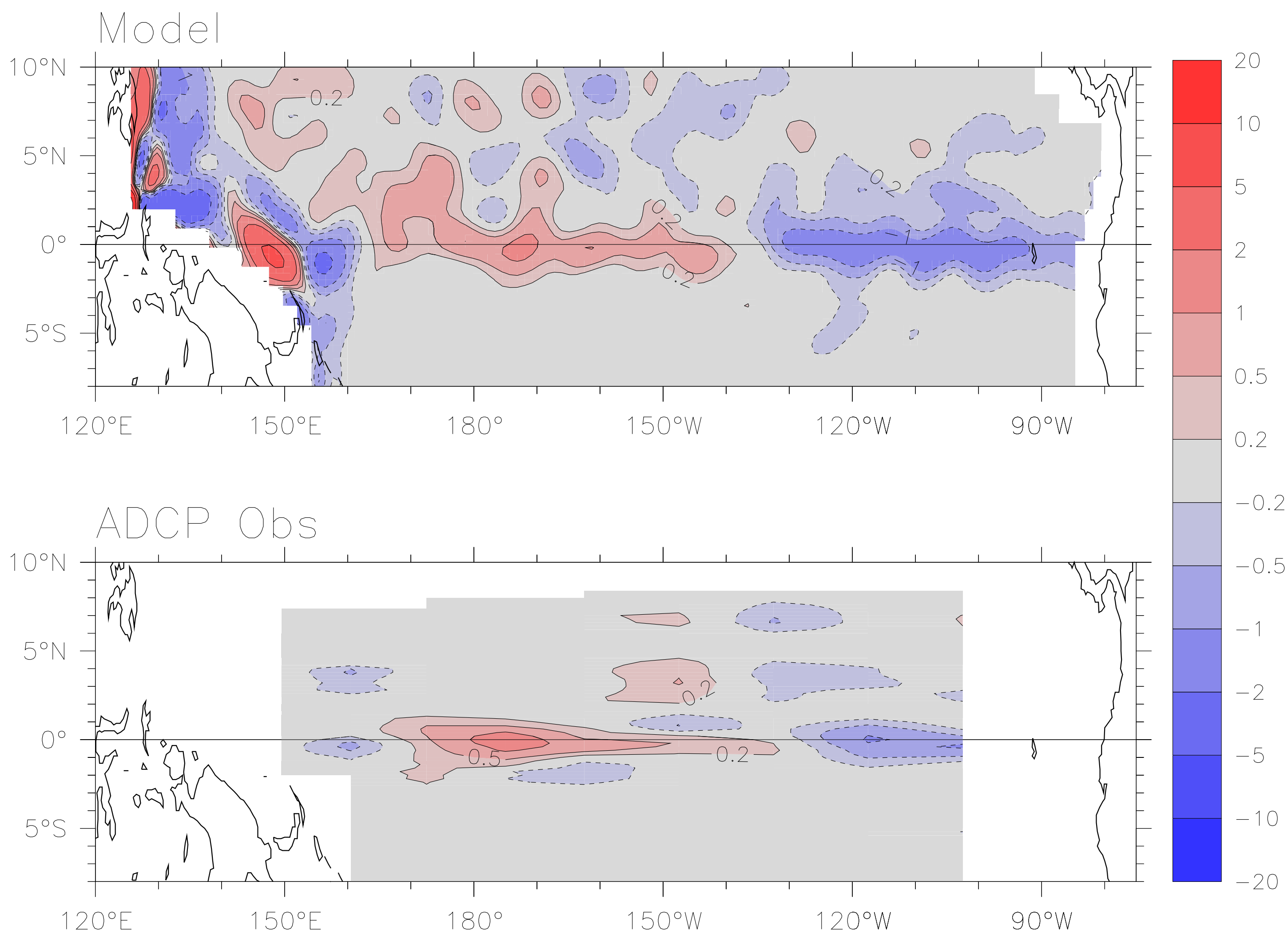
$$\nabla\cdot(\vec{u}\vec{u}h) = \left(\frac{\partial}{\partial x}(uuh) + \frac{\partial}{\partial y}(vuh)\right)\hat{i} + \left(\frac{\partial}{\partial x}(uvh) + \frac{\partial}{\partial y}(vvh)\right)\hat{j}$$

$$= \left(h(uu_x + vu_y) + u(uh_x + vh_y) + uh(u_x + v_y)\right)\hat{i} +$$

$$\left(\underbrace{h(uv_x + vv_y)}_{\text{h}\cdot\text{(simple adv terms)}} + \underbrace{v(uh_x + vh_y) + vh(u_x + v_y)}_{\vec{u}\cdot\text{(continuity terms)}}\right)\hat{j}$$

Mean $\int uu_x \, dz$

G/C model and Johnson ADCP obs (10^{-5} m^2 s^{-2})



Model uu_x is an average over full time dependence. Obs uu_x is from mean u.